STRUCTURE AND EVOLUTION OF VORTEX CORES

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The study of flows with concentrated vorticity is important in the understanding of the dynamics of vortex formation in nature (cyclones, hurricanes, tornados) and in various laboratory devices (centrifugal jets, vortex devices, etc.).

Recently, certain types of vortex flow (ring and tornado-like vortices, vortex strings in a vortex chamber) have been studied in great detail. In particular, the average velocity fields have been measured and the distributions of vorticity have been found (see, e.g., [1-4]).

In the present paper we present new data on the inner structure and evolution of vortex cores. We consider the flow between two coaxial disks of the same radius, rotating with a constant angular velocity in the same direction. The measurements of the average velocity field show that the flow can be considered as a vortex with a rigidly rotating core. By making the flow pattern visible, it can be established that the vortex core cross section loses its circular symmetry when a certain critical Reynolds number is exceeded and takes on the form of an oval, triangle, quadrangle, etc. For large enough Reynolds numbers the core is made up of a system of smaller secondary vortices and the vortex core deforms in a continuous way and exchanges fluid with the surrounding flow field. The exchange occurs via the ejection of spiral arms from the core, which propagate into the outer flow, and via the capture of fluid from the outer flow into the core in the form of separate jets. Analogous structures are observed for other vortex flows.

1. Experiments studying the flow between rotating disks have been carried out in air, water, aqueous solutions of glycerin, and in a solution of tetrachloroethylene in benzene. The disks were placed inside a cylindrical vessel, coaxially aligned with the vessel.

The fundamental parameters, which were observed to qualitatively affect the structure of the flow, were the Reynolds number $Re = \omega R^2/\nu$ and the relative gap width h/R (R is the disk radius, h is the gap between the disks, ω is the angular velocity of the disks, and ν is the kinematic viscosity of the fluid). Re was varied from 10 to $6.4\cdot10^5$, h/R was varied from 0.08 to 1.28. The dimensional parameters were varied between the limits: R = 0.035 to 0.1 m, ω = 0.74 to 314 sec^{-1}, $\nu = 6\cdot10^{-7}$ to $1.2\cdot10^{-3}$ m^2/sec. The radius of the cylindrical vessel was varied from 0.05 to 0.085 m.

The average velocity field between the rotating disks was studied with the help of a laser Doppler anemometer. Experiments were carried out in a solution of tetrachloroethylene in benzene. Light scattering centers were provided by introducing small ebonite particles in the fluid. The density of the solution was chosen to be equal to the density of the particles.

The measurements showed that, in the entire flow region, except for thin boundary layers at the disks, the circular component of the velocity did not depend on the axial coordinate and hence the azimuthal motion of the fluid between the disks was two-dimensional. A typical graph of the circular component of the velocity v_φ along the radial coordinate r is shown in Fig. 1 ($Re = 6.4\cdot10^5$, h/R = 0.5, R = 0.035 m, ω = 314 sec^{-1}, curve 1), where we also show the vorticity $\Omega = v_\varphi/r + \partial v_\varphi/\partial r$ and the circulation $\Gamma = 2\pi r v_\varphi$ calculated from the experimental values of v_φ (curves 2 and 3). It is evident that the flow field consists of a central rigidly rotating core and an outer region in which the vorticity is close to zero. The core radius is less than the radius of the disks, and its angular velocity of rotation is practically equal to the angular velocity of the disks.

Along with the primary (azimuthal) flow between the disks, there is also a secondary flow. The fluid goes into the space between the disks near the center part of the gap, moves inward to the vortex core, and is then ejected outward along the boundary layers at

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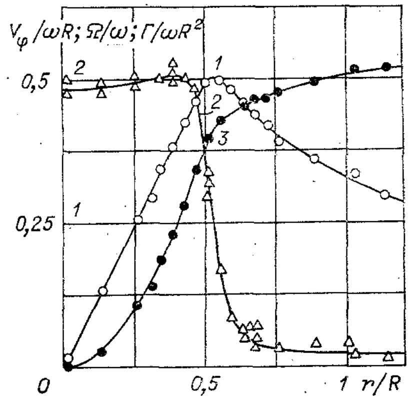

Fig. 1

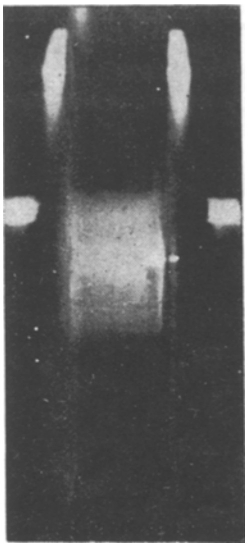

Fig. 2

the surfaces of the disks. The secondary flow velocity is everywhere (except at the boundary layers of the disks) one to two orders of magnitude smaller than the maximum circular flow velocity. More detailed data on the secondary flow outside the core is given in [5].

It follows from the measured results for the average flow velocity between the rotating disks that this flow can be considered as a vortex: the profiles of v_φ, Ω, and Γ, shown in Fig. 1, are characteristic for other vortices as well, such as ring vortices, hurricanes, etc. [1-4, 6, 7].

2. In order to study the inner structure and the evolution of the vortex core, the flow between the rotating disks was made visible by using smoke in air and aluminum powder, dye solutions, and fluorescein in the fluid. The flow patterns were recorded with motion pictures in two different directions: along the axis of rotation of the disks and perpendicular to it. The pictures along the axes of the disks were taken through one of the disks, which was made of a transparent material. The flow field in this case was illuminated using a plane light beam parallel to the plane of the disks.

The flow patterns observed were typical of flows with concentrated vorticity; the dye introduced in the vortex core remained inside the core for a sufficiently long time, even for turbulent flow. This is seen in Fig. 2, where the flow of air between the rotating disks was made visible with the use of dust ($Re = 2 \cdot 10^5$, $h/R = 0.4$, $R = 0.1$ m, $\omega = 314$ sec^{-1}). This behavior of the dye impurity is due to the suppression of turbulence in the core by the rotation and has been considered in detail in [8, 9].

The experiments show that the vortex flow generated in an axially symmetric experimental apparatus does not in general have axial symmetry. The lack of symmetry of the flow is especially evident in the strong deformation of core cross section and in the formation of spiral structures propagating out from the core into the outer flow field.

A series of photographs is shown in Fig. 3 demonstrating the structure of the flow field in a plane parallel to the disks for $h/R = 0.16$ ($R = 0.0625$ m) and for fixed Re. The flow is axisymmetric for small Reynolds numbers ($Re < 400$). For larger values of Re the flow loses its circular symmetry and the cross section of the vortex core successively takes on the shapes of a regular pentangle, quadrangle, triangle, and oval (Fig. 3a-d, respectively, for $Re = 10^3$, $1.5 \cdot 10^3$, $2 \cdot 10^3$, $2.6 \cdot 10^3$, $\omega = 3.7$, 4.8, 6, 10.5 sec^{-1}). Each of the various core shapes exists over a certain range of Re. The transition from one core configuration to another with gradual increase in Re occurs for larger values of Re than the opposite transitions as Re is gradually decreased, i.e., the phenomenon of hysteresis is observed.

For $Re < 2.4 \cdot 10^3$ the flow is laminar. For $2.4 \cdot 10^3 < Re < 3 \cdot 10^3$ a transition regime occurs characterized by smooth transitions from one core shape to another. For $Re > 3 \cdot 10^3$, the flow becomes turbulent.

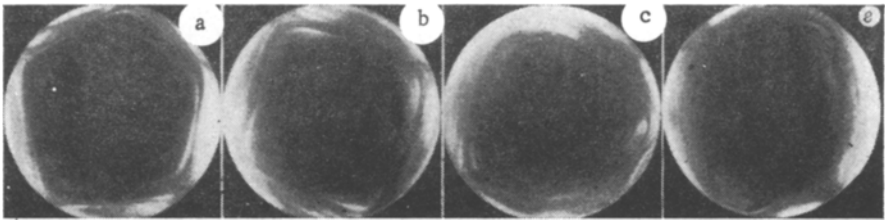

Fig. 3

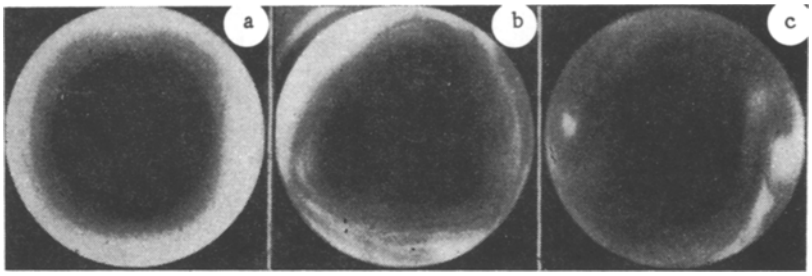

Fig. 4

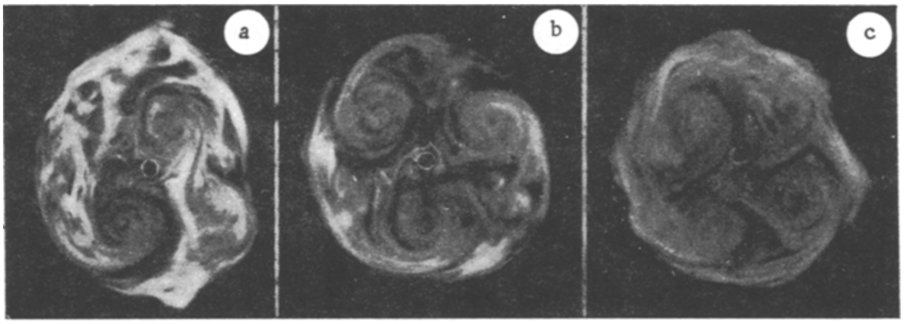

Fig. 5

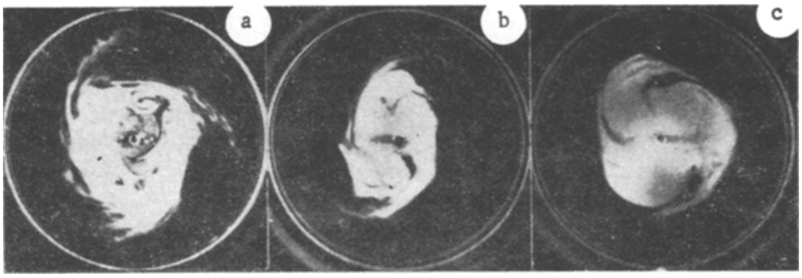

Fig. 6

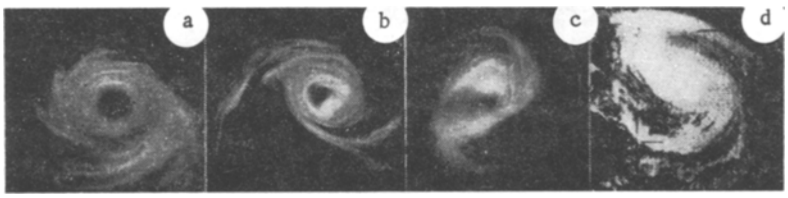

Fig. 7

In the laminar regime the number of figures formed with change of Re is smaller, the larger the value of the gap h/R between the disks. For example, when h/R = 0.32, the circular shape of the core cross section (Fig. 4a, Re = $2.6 \cdot 10^2$, R = 0.0625 m, ω = 0.74 $\sec^{-1}$) transforms directly into the triangular shape (Fig. 4b, Re = $1.1 \cdot 10^3$, ω = 3.5 $\sec^{-1}$), and then into the oval shape for Re = $1.3 \cdot 10^3$. When h/R = 1.28, the circular core transforms directly into the oval shape (Fig. 4c, Re = $9 \cdot 10^2$, ω = 4.5 $\sec^{-1}$). The polygonal curve bounding the vortex core itself rotates with a constant angular velocity n which is smaller than the angular velocity of the disks. It is known [10, 11] that for plane flow of an ideal fluid with elliptical and polygonal regions of constant vorticity Ω

$$n = \Omega ab/(a + b)^2, \; n \approx \Omega(k - 1)/2k,$$

where a and b are the semimajor and semiminor axes of the ellipse, and k is the number of corners of the polygon. The measured value of n is always less than the value predicted by the above formulas with $\Omega = 2\omega$, and the relative error can reach 30% in some cases.

In the turbulent flow regime there are the following characteristic features.

1. As in the transition flow regime, the cortex core continuously deforms. But different shapes of the core cross section (oval, triangle, quadrangle, etc.) are observed at different instants of time, and the number of such figures is much larger than in the cases of laminar or transition flow for the same value of h/R.

2. The vortex core is made up of a system of smaller (secondary) vortices rotating in the same direction as the primary vortex. The distribution of the secondary vortices is the most symmetric when the shape of the core cross section is close to a circle. When the core deforms, the secondary vortices are deformed and destroyed, and new secondary vortices are generated. When the circular shape of the core cross section is restored, its structure is also restored. Figure 5 shows a vortex core made up of (at different instants of time), two (a), three (b), and four (c) secondary vortices (Re = $4.9 \cdot 10^4$, h/R = 0.48, R = 0.0625 m, ω = 12.5 $\sec^{-1}$). The centers of the secondary vortices rotate about the axis of the disks with the same angular velocity as the disks. The vorticity inside the secondary vortices shown in Fig. 5 is about 1.2 times larger than twice the angular velocity of the disks.

As h/R decreases, the number of secondary vortices in the core increases and their distribution and structure become disordered.

3. The vortex core regularly exchanges fluid with the surrounding flow. The exchange occurs via the ejection of spiral arms from the core and the capture of the outer fluid into the core in the form of separate jets.

Ejection of fluid from the core usually occurs when the core cross section transforms from a noncircular shape to a circle. The process of ejection of a given arm lasts for several revolutions of the disks. The frequency and intensity of the ejections increases as h/R decreases. For very intense ejections the fluid in the arm moves beyond the edges of the disks. Figure 6a, b shows spiral arms breaking away from the corners of a triangular vortex core (Re = $4.9 \cdot 10^4$, h/R = 0.32, R = 0.0625 m, ω = 12.5 $\sec^{-1}$). and from an oval core (Re = $4.9 \cdot 10^4$, h/R = 0.48, R = 0.0625 m, ω = 12.5 $\sec^{-1}$).

The capture of fluid into the core from the outer flow occurs by a "winding" of fluid into the secondary vortices. This process is seen in Fig. 6c, where one can trace three dark jets of noncolored fluid going into the vortex core (Re = $4.9 \cdot 10^4$, h/R = 0.48, R = 0.0625 m, ω = 12.5 $\sec^{-1}$). The intensity of capture increases when the core is deformed.

It should be pointed out that the flow patterns shown in Figs. 3-6 do not depend upon the axial coordinate.

3. The features observed here of the structure and evolution of the vortex core in the flow between rotating disks are also observed in other vortex flows. A series of experiments were performed in which the vortex core in other physical conditions was studied. In Fig. 7a-c, we show the cross section of the core for different vortices: an air ring vortex; a vortex formed from rotating water vapor floating in air; and a vortex string in a vortex chamber. Figure 7d shows a photograph of an atmospheric hurricane taken by the satellite Tairos III (1961). The spiral arms and the nonaxisymmetric oval shape of the vortex cores are clearly visible on the photographs. Triangle, quadrangle, and pentangle

shapes for the core cross section are observed in a series of cases for ring vortices and vortices formed in a vortex chamber. Large-scale atmospheric cyclone vortices also have spiral structure. It is possible that the examples presented here might also be relevant for rotating spiral galaxies.

One can assume that the observed commonality in the structures of vortex cores is not an accident, but is due to certain common causes. The study of these causes and the conditions for the formation and existence of nonaxisymmetric vortex structures requires further study.

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